Homework 8: Due April 2 (11:59 p.m.)

Instructions

- **Answer each question on a separate page.**
- **Honors questions are optional. They will not count towards your grade in the course. However you are encouraged to submit your solutions to these problems to receive feedback on your attempts. Our estimation of the difficulty level of these problems is expressed through an indicative number of stars** $('*' =$ **easiest**$)$ **to** $('*****' =$ **hardest**$)$.
- **You must enter the names of your collaborators or other sources as a response to Question** 0**. Do NOT leave this blank; if you worked on the homework entirely on your own, please write "None" here. Even though collaborations in groups of up to** 3 **people are encouraged, you are required to write your own solution.**

Question 0: List all your collaborators and sources: ($-\infty$ points if left blank)

Question 1: Huffman Codes (5+5=10 points)

1. In general, are Huffman codes unique? That is, given a set of letters and their corresponding frequencies, does any Huffman code-generating *algorithm* output the same encoding scheme? If yes, justify. Otherwise provide a counterexample. (Hint: different letters may have the same frequency in the general case.)
2. Is it possible that in an *optimal* code, a letter with lower frequency has a shorter encoding than a letter with a higher frequency? If no, explain why not. Otherwise, provide an explicit example of such code.

Question 2: (10 points)

Consider a file that uses the following list of symbols with the corresponding frequencies:

Letter	A	B	C	D	E	F	G
Frequency	0.06	0.09	0.10	0.12	0.15	0.16	0.32

Find an optimal prefix code based on Huffman's algorithm (using the symbols 0 and 1 only). Describe both the code (i.e., mapping from symbols to bit strings) and the corresponding tree.

Question 3: (5 points)

Prove that in any undirected graph, the sum of the degrees of all the vertices is an even number. (Recall that the degree of each vertex is just the number of edges that include it. For example, the degree of vertex B in Figure 1 is 3.)

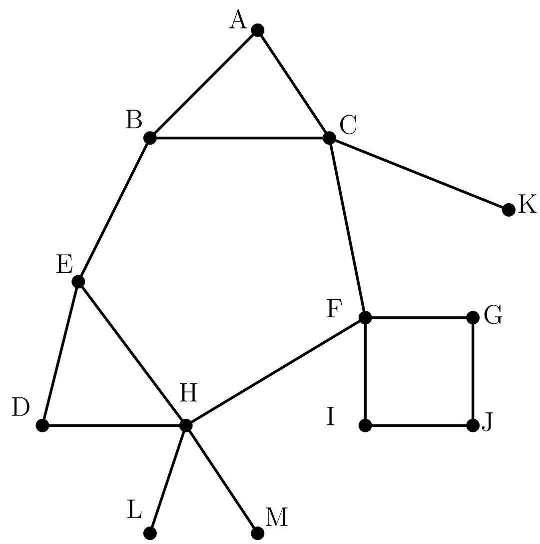

Figure 1: Graph to traverse

Question 4: (5+5=10 points)

Consider the undirected graph shown in Figure 1. In what order are the nodes visited (i.e., colored in *gray*) in each case? Fill in the missing entries for Table 1. Assume that we use alphabetical order to break ties. For example, from A we will visit B before C, and so on.

1. BFS traversal starting at node A.
2. DFS traversal starting at node A.

Traversal	1	2	3	4	5	6	7	8	9	10	11	12	13
BFS	A	B											
DFS	A	B											

Table 1: Fill in the order of visited nodes for each graph traversal method.

Question 5: (10+5=15 points)

Recall that a *cycle* in an undirected graph is a sequence of distinct vertices $(v_1, v_2, \ldots, v_k)$ with $k \geq 3$ such that the edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots, \{v_{k-1}, v_k\}$ and also $\{v_k, v_1\}$ all exist in the edge set. For example, in Figure 1 (A, B, C) and (F, G, J, I) form a cycle.

1. Design an algorithm which, given an undirected graph, determines whether the graph has a cycle. If the graph has $|V|$ vertices and $|E|$ edges, your algorithm should run in $O(|V| + |E|)$ time.
2. Justify the correctness and run-time of your algorithm.

Honors Questions (Optional)

Question 6: Honors (0 points)

(**) Generalize Huffman's algorithm to ternary codes (i.e., using the symbols 0, 1, and 2 instead of just 0 and 1), and prove that it yields an optimal code.